

# UNAVOIDABLE CONFLICT BETWEEN MASSIVE GRAVITY MODELS AND MASSIVE TOPOLOGICAL TERMS

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Massive gravity models in 2+1 dimensions, such as those obtained by adding to Einstein's gravity the usual Fierz-Pauli, or the more complicated Ricci scalar squared ( $R^2$ ), terms, are tree level unitary. Interesting enough these seemingly harmless systems have their unitarity spoiled when they are augmented by a Chern-Simons term. Furthermore, if the massive topological term is added to  $R + R_{\mu\nu}^2$  gravity, or to  $R + R_{\mu\nu}^2 + R^2$  gravity (higher-derivative gravity), which are nonunitary at the tree level, the resulting models remain nonunitary. Therefore, unlike the common belief, as well as the claims in the literature, the coexistence between three-dimensional massive gravity models and massive topological terms is conflicting.

*Keywords:* massive gravity models; topological terms; tree unitarity.

The remarkable properties of topological tensor gauge theories in  $2 + 1$  dimensions are by now not only well-appreciated but also well-understood. The linearized versions of these models describe single massive but gauge-invariant excitations<sup>1</sup>. Nonetheless, according to a somewhat obscure tree unitarity lore it is expected that the operation of augmenting a nontopological massive gravity model through the topological term would transform the nonunitary systems into unitary ones and preserve the tree unitarity of the originally unitary models. In truth, this addition does more harm than good. Indeed, innocuous avowedly tree unitary models, such as Fierz-Pauli gravity or the more sophisticated  $R + R^2$  gravity, become nonunitary after the topological addition, while admittedly nonunitary systems, such as  $R + R_{\mu\nu}^2$  gravity or higher-derivative gravity ( $R + R^2 + R_{\mu\nu}^2$ ) remain stubbornly nonunitary after the topological enlargement.

Our aim here is to discuss the incompatibility between massive gravity models and massive topological terms. The analysis comprises massive topological gravity systems that are focus of the controversy in the literature: topological Fierz-Pauli gravity and topological higher-derivative gravity—they are wrongly considered as tree unitary models<sup>2–4</sup>—and topological  $R + R^2$  and  $R + R_{\mu\nu}^2$  gravity. We will show that these topological models are without exception nonunitary at the tree level.

To probe the tree unitarity of the models we make use of the method that consists in saturating the propagator with external conserved currents,  $T^{\mu\nu}$ , compatible with the symmetries of the theory. The unitarity analysis is based on the residues of the saturated propagator ( $SP$ ): the tree unitarity is ensured if the residue at each pole of the  $SP$  is positive. Note that the  $SP$  is nothing but the current-current amplitude in momentum space.

Natural units are used throughout. Our signature is  $(+, -, -)$ . The Riemann and Ricci tensors are defined respectively as  $R^\rho_{\lambda\mu\nu} = -\partial_\nu\Gamma^\rho_{\lambda\mu} + \partial_\mu\Gamma^\rho_{\lambda\nu} - \Gamma^\sigma_{\lambda\mu}\Gamma^\rho_{\sigma\nu} + \Gamma^\sigma_{\lambda\nu}\Gamma^\rho_{\sigma\mu}$  and  $R_{\mu\nu} = R^\rho_{\mu\nu\rho}$ .

We consider first topological Fierz-Pauli gravity (TFPG). The Lagrangian related to this model is the sum of Einstein, standard Fierz-Pauli, and Chern-Simons, terms, namely,

$$\mathcal{L} = a \frac{2}{\bar{\kappa}^2} \sqrt{g} R - \frac{m^2}{2} (h_{\mu\nu}^2 - h^2) + \frac{1}{\mu} \epsilon^{\lambda\mu\nu} \Gamma^\rho_{\lambda\sigma} \left( \partial_\mu \Gamma^\sigma_{\rho\nu} + \frac{2}{3} \Gamma^\sigma_{\mu\beta} \Gamma^\beta_{\nu\rho} \right), \quad (1)$$

at quadratic order in  $\bar{\kappa}$ , where  $\bar{\kappa}^2$  is a suitable constant that in four dimensions

is equal to  $24\pi G$ , with  $G$  being Newton's constant<sup>5</sup>. Here  $g_{\mu\nu} \equiv \eta_{\mu\nu} + \bar{\kappa} h_{\mu\nu}$ ,  $h \equiv \eta_{\mu\nu} h^{\mu\nu}$ , and  $a$  is a convenient parameter that can take the values  $+1$  (Einstein's term with the usual sign) or  $-1$  (Einstein's term with the “wrong” sign), so that this is the most general such model. From now on indices are raised (lowered) with  $\eta^{\mu\nu}$  ( $\eta_{\mu\nu}$ ).

To compute the SP we have to find beforehand the propagator, which involves much algebra. However, the calculations are greatly simplified if we appeal to a set of operators made up by the usual three-dimensional Barnes-Rivers operators<sup>6</sup>, i.e.,

$$P_{\mu\nu, \rho\sigma}^1 = \frac{1}{2} (\theta_{\mu\rho} \omega_{\nu\sigma} + \theta_{\mu\sigma} \omega_{\nu\rho} + \theta_{\nu\rho} \omega_{\mu\sigma} + \theta_{\nu\sigma} \omega_{\mu\rho}),$$

$$P_{\mu\nu, \rho\sigma}^2 = \frac{1}{2} (\theta_{\mu\rho} \theta_{\nu\sigma} + \theta_{\mu\sigma} \theta_{\nu\rho} - \theta_{\mu\nu} \theta_{\rho\sigma}),$$

$$P_{\mu\nu, \rho\sigma}^0 = \frac{1}{2} \theta_{\mu\nu} \theta_{\rho\sigma}, \quad \bar{P}_{\mu\nu, \rho\sigma}^0 = \omega_{\mu\nu} \omega_{\rho\sigma},$$

$$\bar{\bar{P}}_{\mu\nu, \rho\sigma}^0 = \theta_{\mu\nu} \omega_{\rho\sigma} + \omega_{\mu\nu} \theta_{\rho\sigma},$$

where  $\theta_{\mu\nu}$  and  $\omega_{\mu\nu}$  are the well-known transverse and longitudinal vector projector operators  $\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\square}$ ,  $\omega_{\mu\nu} = \frac{\partial_\mu \partial_\nu}{\square}$ , and the operator

$$P_{\mu\nu, \rho\sigma} \equiv \frac{\square \partial^\lambda}{4} [\epsilon_{\mu\lambda\rho} \theta_{\nu\sigma} + \epsilon_{\mu\lambda\sigma} \theta_{\nu\rho} + \epsilon_{\nu\lambda\rho} \theta_{\mu\sigma} + \epsilon_{\nu\lambda\sigma} \theta_{\mu\rho}],$$

which has its origin in the linearization of the Chern-Simons term, i.e.,

$$\mathcal{L}_{C.S.lin} = \frac{1}{2} \frac{1}{M} h^{\mu\nu} P_{\mu\nu, \rho\sigma} h^{\rho\sigma},$$

where  $M \equiv \mu/\bar{\kappa}^2$ . The corresponding multiplicative table is displayed in Table 1.

Adapting to  $2+1$  dimensions, with the help of the data from Table 1, the algorithm for calculating the propagator related to four-dimensional gravity theories<sup>7</sup> and using the resulting prescription, we find that the propagator for TFPG assumes the form

$$\begin{aligned} O^{-1} = & -\frac{1}{m^2}P^1 - \frac{M^2(m^2 + a\Box)}{\Box^3 + M^2a^2\Box^2 + 2am^2M^2\Box + M^2m^4}P^2 - \frac{m^2 + a\Box}{2m^4}\overline{P}^0 \\ & + \frac{1}{2m^2}\overline{\overline{P}}^0 - \frac{M}{\Box^3 + M^2a^2\Box^2 + 2am^2M^2\Box + M^2m^4}P. \end{aligned}$$

Consequently, the saturated propagator is given by

$$SP_{TFPG} = T^{\mu\nu}O_{\mu\nu, \rho\sigma}^{-1}T^{\rho\sigma}.$$

Performing the computations, we promptly obtain in momentum space

$$SP_{TFPG} = \left[ T^{\mu\nu}(k)T_{\mu\nu}(k) - \frac{1}{2}T^2(k) \right] \frac{M^2(m^2 - k^2a)}{k^6 - M^2a^2k^4 + 2am^2M^2k^2 - M^2m^4}.$$

At first sight it seems that we should start our analysis by setting  $a = -1$  since in the limit  $m^2 = 0$  (1) reduces to pure massive topological gravity (MTG)—a theory that requires  $a = -1$  to be ghost-free<sup>1</sup>. Nevertheless, a straightforward numerical computation shows that among the roots of the equation

$$x^3 - a^2\lambda m^2x^2 + 2a\lambda m^4x - \lambda m^6 = 0, \quad (2)$$

where  $x \equiv k^2$ ,  $a = -1$  and  $\lambda \equiv \left(\frac{M}{m}\right)^2$ , there are always two complex roots for any positive  $\lambda$  value. Therefore, this model is unphysical and must be rejected.

Now we shall concentrate our attention on the system with  $a = +1$ . In this case the roots of (2) can be classified as

$\lambda < 27/4$	one real root and two complex ones
$\lambda = 27/4$	three real roots: $x_1 = x_2 = 4x_3 = 4M^2/9$
$\lambda > 27/4$	three distinct positive real roots

Accordingly, the viability of the theory requires  $\lambda > \frac{27}{4}$ , which implies that the  $SP_{TFPG}$  may be written as

$$SP_{TFPG} = X_1 + X_2 + X_3,$$

where

$$X_1 = \frac{M^2(m^2 - x_1)F(k)}{(x_1 - x_2)(x_1 - x_3)(k^2 - x_1)}, \quad X_2 = \frac{M^2(m^2 - x_2)F(k)}{(x_2 - x_1)(x_2 - x_3)(k^2 - x_2)},$$

$$X_3 = \frac{M^2(m^2 - x_3)F(k)}{(x_3 - x_1)(x_3 - x_2)(k^2 - x_3)}.$$

with  $F(k) \equiv T^{\mu\nu}(k)T_{\mu\nu}(k) - \frac{1}{2}T^2(k)$ . We are assuming without any loss of generality that  $x_1 > x_2 > x_3$ .

Let us then find a suitable basis for expanding the sources. The class of independent vectors in momentum space ,

$$k^\mu \equiv (k^0, \mathbf{k}) , \quad \tilde{k}^\mu \equiv (k^0, -\mathbf{k}) , \quad \varepsilon^\mu \equiv (0, \vec{\epsilon}) ,$$

where  $\vec{\epsilon}$  is a unit vector orthogonal to  $\mathbf{k}$ , does the job. In this basis the symmetric current tensor assumes the form

$$T^{\mu\nu} = Ak^\mu k^\nu + B\tilde{k}^\mu \tilde{k}^\nu + C\varepsilon^\mu \varepsilon^\nu + Dk^{(\mu} \tilde{k}^{\nu)} + Ek^{(\mu} \varepsilon^{\nu)} + F\tilde{k}^{(\mu} \varepsilon^{\nu)}.$$

As a consequence,

$$F(k) = \frac{1}{2}[(A - B)k^2]^2 + \frac{C^2}{2} + \frac{k^2}{2}(E^2 - F^2) - (A - B)k^2 C.$$

Assuming, as usual, that  $T \geq 0$ , we get  $C \leq 0$ , implying  $F(k) > 0$ . If  $x_1 < m^2$ ,  $\text{Res } SP_{TFPG}|_{k^2=x_1} > 0$ ,  $\text{Res } SP_{TFPG}|_{k^2=x_2} < 0$  and  $\text{Res } SP_{TFPG}|_{k^2=x_3} > 0$ .

The theory is causal and has two spin-2 physical particles of masses equal to  $x_1$  and  $x_3$ , respectively, and one spin-2 ghost of mass  $x_2$ . On the other hand, if  $x_1 > m^2$ , the model is causal as well and has at least one spin-2 ghost of mass  $x_1$ . These models are thus nonunitary at the tree level due to the presence of the ghosts. Note that if we set  $M^2 = \infty$ , we recover pure Fierz-Pauli gravity (FPG). In this case  $SP_{FPG} = \frac{F(k)}{k^2 - m^2}$  and  $\text{Res } SP_{FPG}|_{k^2=m^2} > 0$ . Since the residue of the  $SP$  is positive, FPG is tree unitary—a well-known result. Therefore, the topological term is responsible for breaking down the tree unitarity of the harmless FPG.

We discuss in the following the claims in the literature<sup>2,3</sup> concerning the tree unitarity of TFPG with  $a = +1$  and  $\lambda > \frac{27}{4}$ . The authors of Refs. 2 and 3 simply affirm that the model in hand is tree unitary; however, no explicit proof is presented to give support to their statement. For clarity's sake, we quote from Refs. 2 and 3, in this order: “It can be checked that the condition  $\lambda > \frac{27}{4}$  must be fulfilled in order that poles that correspond to tachyons and ghosts be suppressed from the spectrum.” “It is checked that tachyons and ghosts are excluded from the spectrum whenever  $\lambda > \frac{27}{4}$ .” We have shown in detail that this is not true. Actually, the authors of these works have made a mistake as far as the analysis of the sign of the residues of  $SP_{TFPG}$  is concerned, which led them to conclude incorrectly that TFPG is unitary at the tree level.

We consider now topological higher-derivative gravity (THDG), whose Lagrangian has the form

$$\mathcal{L} = \sqrt{-g} \left( a \frac{2R}{\kappa^2} + \frac{\alpha}{2} R^2 + \frac{\beta}{2} R_{\mu\nu}^2 \right) + \frac{1}{\mu} \epsilon^{\lambda\mu\nu} \Gamma^\rho_{\lambda\sigma} \left( \partial_\mu \Gamma^\sigma_{\rho\nu} + \frac{2}{3} \Gamma^\sigma_{\mu\beta} \Gamma^\beta_{\nu\rho} \right), \quad (3)$$

where  $\kappa^2$  is a constant that in four dimensions is equal to  $32\pi G$ . Here  $\alpha$  and  $\beta$  are suitable dimensional constants. Before going on we must answer a crucial question: What is the use of augmenting pure three-dimensional gravity through the quadratic terms  $R^2$  and  $R_{\mu\nu}^2$ ? The answer is quite straightforward: The quadratic terms convert Einstein's gravity, that is trivial from the

Table 1: Multiplicative operator algebra fulfilled by  $P^1$ ,  $P^2$ ,  $P^0$ ,  $\overline{P}^0$ ,  $\overline{\overline{P}}^0$  and  $P$ . Here  $P^{\theta\omega}_{\mu\nu, \rho\sigma} \equiv \theta_{\mu\nu}\omega_{\rho\sigma}$  and  $P^{\omega\theta}_{\mu\nu, \rho\sigma} \equiv \omega_{\mu\nu}\theta_{\rho\sigma}$ .

	$P^1$	$P^2$	$P^0$	$\overline{P}^0$	$\overline{\overline{P}}^0$	$P$
$P^1$	$P^1$	0	0	0	0	0
$P^2$	0	$P^2$	0	0	0	$P$
$P^0$	0	0	$P^0$	0	$P^{\theta\omega}$	0
$\overline{P}^0$	0	0	0	$\overline{P}^0$	$P^{\omega\theta}$	0
$\overline{\overline{P}}^0$	0	0	$P^{\omega\theta}$	$P^{\theta\omega}$	$2(P^0 + \overline{P}^0)$	0
$P$	0	$P$	0	0	0	$-\square^3 P^2$

classical viewpoint, into a nontrivial model; furthermore, within the quantum scheme, higher-derivative gravity (HDG), unlike pure Einstein's gravity, has propagating degrees of freedom. In other words, the net effect of adding quadratic or topological terms to pure Einstein's gravity is just the same: to produce a nontrivial gravity model with gravitons. HDG models have interesting properties of their own:

- (i) The general solution of its linearized version<sup>8</sup> great resembles, *mutatis mutantis*, the four-dimensional metric of a straight  $U(1)$  gauge cosmic string in the context of linearized four-dimensional HDG<sup>9</sup>.
- (ii) Contrary to what happens with the Newtonian potential—which has a logarithmic singularity at the origin and is unbounded at infinity—HDG's nonrelativistic potential is extremely well-behaved: it is finite at the origin and zero at infinity<sup>10</sup>.
- (iii) Both antigravity and gravitational shielding can coexist without conflict with HDG<sup>11</sup>.
- (iv) The gravitational deflection angle associated to HDG is always less than that related to pure gravity<sup>8</sup>.

Despite these nice properties, HDG possesses a ghost pole in the tree propagator which renders it nonunitary within the standard perturbation scheme. However, according to the already mentioned tree unitarity lore, it is naively

expected that if we augment HDG via the Chern-Simons term we would arrive at a tree unitary theory<sup>4</sup>. Our main objective in what follows is to expose the fallacy of this conjecture.

To find the propagator concerning THDG, we linearize (3) and add to the result the gauge-fixing Lagrangian,  $\mathcal{L}_{g.f.} = \frac{1}{2\lambda} (h_{\mu\nu}{}^{,\nu} - h_{,\mu})^2$ , that corresponds to the de Donder gauge. The propagator is given by

$$\begin{aligned} \mathcal{O}^{-1} = & \frac{2\lambda}{k^2} P^1 - \frac{2M^2(2a - b\Box)}{\Box[M^2b^2\Box^2 - 4(abM^2 - 1)\Box + 4M^2a^2]} P^2 \\ & + \frac{1}{\Box[a + b(\frac{3}{2} + 4c)\Box]} P^0 + \left[ -\frac{4\lambda}{\Box} + \frac{2}{\Box[a + b(\frac{3}{2} + 4c)\Box]} \right] \bar{P}^0 \\ & + \frac{1}{\Box[a + b(\frac{3}{2} + 4c)\Box]} \bar{\bar{P}}^0 + \frac{4M}{\Box[M^2b^2\Box^2 - 4(abM^2 - 1)\Box + 4M^2a^2]} P, \end{aligned}$$

where  $b \equiv \frac{\beta\kappa^2}{2}$  and  $c \equiv \frac{\alpha}{\beta}$ .

In momentum space, the associated  $SP$  assumes the form:

$$\begin{aligned} SP_{THDG} = & (T^{\mu\nu}T_{\mu\nu} - \frac{1}{2}T^2) \left[ -\frac{1 + \sqrt{1 - 2abM^2}}{2a\sqrt{1 - 2abM^2}(k^2 - M_1^2)} \right] \\ & + (T^{\mu\nu}T_{\mu\nu} - \frac{1}{2}T^2) \left[ -\frac{-1 + \sqrt{1 - 2abM^2}}{2a\sqrt{1 - 2abM^2}(k^2 - M_2^2)} \right] \\ & + \frac{T^{\mu\nu}T_{\mu\nu} - T^2}{ak^2} + \frac{\frac{1}{2}T^2}{a(k^2 - m^2)}, \end{aligned} \quad (4)$$

where

$$\begin{aligned} M_1^2 & \equiv \left( \frac{2}{b^2M^2} \right) [1 - abM^2 - \sqrt{1 - 2abM^2}], \\ M_2^2 & \equiv \left( \frac{2}{b^2M^2} \right) [1 - abM^2 + \sqrt{1 - 2abM^2}], \\ m^2 & \equiv \frac{a}{b(\frac{3}{2} + 4c)}. \end{aligned}$$

We are now ready to analyse the excitations and mass counts for generic signs and values of the parameters and for both allowed signs of  $a$ . To begin with, we set  $a = -1$ . The absence of tachyons in the dynamical field requires  $b > 0$  and  $\frac{3}{2} + 4c < 0$  or  $-\frac{1}{2} < bM^2 < 0$  and  $\frac{3}{2} + 4c > 0$ .



The former leads to  $\text{Res } SP_{THDG}|_{k^2=M_1^2} > 0$ ,  $\text{Res } SP_{THDG}|_{k^2=M_2^2} > 0$ ,  $\text{Res } SP_{THDG}|_{k^2=0} = 0$  and  $\text{Res } SP_{THDG}|_{k^2=m^2} < 0$ , while the latter results in  $\text{Res } SP_{THDG}|_{k^2=M_1^2} > 0$ ,  $\text{Res } SP_{THDG}|_{k^2=M_2^2} < 0$ ,  $\text{Res } SP_{THDG}|_{k^2=0} = 0$  and  $\text{Res } SP_{THDG}|_{k^2=m^2} < 0$ . The particle content related to the first situation is two massive spin-2 physical particles, one massless spin-2 nonpropagating particle and one massive spin-0 ghost, whereas that concerning the second one is one massive spin-2 physical particle, one massive spin-2 ghost, one nonpropagating graviton and one massive spin-0 ghost. Therefore, unlike the claim in the literature<sup>4</sup>, THDG with Einstein's term with the “wrong” sign is tree nonunitary. Note that the authors of Ref. 4 state that “the spin-0 sector displays a massless pole along with massive poles” as well as that “the massive gravitons propagate as in the pure Einstein-Chern-Simons model: negative-norm states do not appear that spoil the spectrum, which does not affect the unitarity”, pure and simple. A quick glimpse at Table 2 it is sufficient to convince anyone of the wrongness of these affirmations. In truth, the authors of this work made a mistake in examining both the excitations and mass counts of THDG with  $a = -1$ .

Could it be that if we have chosen  $a = +1$  we would have arrived at an unitary system? The response to this question is negative. Indeed, assuming (i)  $b < 0$  and  $\frac{3}{2} + 4c < 0$  or (ii)  $0 < bM^2 < \frac{1}{2}$  and  $\frac{3}{2} + 4c > 0$  in order to get rid of the tachyions, we come to the conclusion that (i)  $\text{Res } SP_{THDG}|_{k^2=M_1^2} < 0$ ,  $\text{Res } SP_{THDG}|_{k^2=M_2^2} < 0$ ,  $\text{Res } SP_{THDG}|_{k^2=0} = 0$  and  $\text{Res } SP_{THDG}|_{k^2=m^2} > 0$ , and (ii)  $\text{Res } SP_{THDG}|_{k^2=M_1^2} < 0$ ,  $\text{Res } SP_{THDG}|_{k^2=M_2^2} > 0$ ,  $\text{Res } SP_{THDG}|_{k^2=0} = 0$  and  $\text{Res } SP_{THDG}|_{k^2=m^2} > 0$ , implying that the model has (i) two massive spin-2 ghosts, one massless spin-2 nonpropagating particle and one massive spin-0 physical particle or (ii) one massive spin-2 ghost, one massive spin-2 physical particle, one nonpropagating graviton and one massive spin-0 physical particle. These systems, as the preceding ones, are also nonunitary at the tree level. The above is summarized in Table 2. The remaining systems are tachyonic.

We discuss now the tree unitarity of the models obtained from THDG by judiciously choosing the parameters  $\alpha$ ,  $\beta$  and  $M^2$  as well as the signs of  $a$ .

- *Pure Massive Topological Gravity* ( $\alpha = \beta = 0$ )

$$SP = -\frac{T^{\mu\nu}T_{\mu\nu} - \frac{1}{2}T^2}{a(k^2 - a^2M^2)} + \frac{T^{\mu\nu}T_{\mu\nu} - T^2}{ak^2}$$

Table 2: Unitarity analysis of topological higher-derivative gravity

$a$	$b$	$\frac{3}{2} + 4c$	excitations and mass counts	tachyons	unitarity
-1	$> 0$	$< 0$	2 massive spin-2 normal particles 1 massless spin-2 nonpropagating particle 1 massive spin-0 ghost	no one	nonunitary at the tree level
-1	$\frac{-1}{2M^2} < b < 0$	$> 0$	1 massive spin-2 normal particle 1 massless spin-2 nonpropagating particle 1 massive spin-2 ghost 1 massive spin-0 ghost	no one	nonunitary at the tree level
+1	$< 0$	$< 0$	2 massive spin-2 ghosts 1 massless spin-2 nonpropagating particle 1 massive spin-0 normal particle	no one	nonunitary at the tree level
+1	$0 < b < \frac{1}{2M^2}$	$> 0$	1 massive spin-2 normal particle 1 massless spin-2 nonpropagating particle 1 massive spin-2 ghost 1 massive spin-0 normal particle	no one	nonunitary at the tree level

$a = -1$ : One massive spin-2 physical particle and one nonpropagating graviton; the model is nontachyonic and tree unitary.

$a = +1$ : One massive spin-2 ghost and one nonpropagating graviton; the system is nontachyonic and nonunitary at the tree level.

*Comment*: Pure massive topological gravity is unitary if and only if the sign of the Einstein's term is chosen to be  $-1$ . This result was obtained in Ref. 1 using a quite different approach.

- *Pure  $R + R_{\mu\nu}^2$  Gravity* ( $\alpha = 0, M^2 = \infty$ )

$$SP = -\frac{T^{\mu\nu}T_{\mu\nu} - \frac{1}{2}T^2}{a(k^2 + \frac{2a}{b})} + \frac{T^{\mu\nu}T_{\mu\nu} - T^2}{ak^2} + \frac{\frac{1}{2}T^2}{a(k^2 - \frac{3}{2}b)}$$

$a = -1$  and  $b > 0$ : One spin-2 physical particle of mass  $m_2 = \sqrt{\frac{2}{b}}$ , one nonpropagating graviton and one spin-0 ghost of mass  $m_0 = \sqrt{\frac{3b}{2}}$ ; the model is nontachyonic and tree nonunitary.

The remaining models are unphysical because they have complex masses.

*Comment*: Pure  $R + R_{\mu\nu}^2$  gravity possesses no tachyons if and only if  $a = -1$

(Einstein's term with the “wrong” sign) and  $b > 0$ . Nonetheless, the model has a massive scalar ghost which renders it nonunitary at the tree level.

- *Topological  $R + R_{\mu\nu}^2$  Gravity* ( $\alpha = 0$ )

$$\begin{aligned}
SP = & (T^{\mu\nu}T_{\mu\nu} - \frac{1}{2}T^2) \left[ -\frac{1 + \sqrt{1 - 2abM^2}}{2a\sqrt{1 - 2abM^2}(k^2 - M_1^2)} \right] \\
& + (T^{\mu\nu}T_{\mu\nu} - \frac{1}{2}T^2) \left[ -\frac{-1 + \sqrt{1 - 2abM^2}}{2a\sqrt{1 - 2abM^2}(k^2 - M_2^2)} \right] \\
& + \frac{T^{\mu\nu}T_{\mu\nu} - T^2}{ak^2} + \frac{\frac{1}{2}T^2}{a(k^2 - \frac{3}{2}b)}
\end{aligned}$$

$a = -1$  and  $b > 0$ : Two spin-2 physical particles of masses equal to  $M_1$  and  $M_2$ , one propagating graviton and one spin-0 ghost of mass  $m_0 = \sqrt{\frac{3b}{2}}$ ; the system is nontachyonic and tree nonunitary.

$a = +1$  and  $0 < b < \frac{1}{2M^2}$ : One spin-2 ghost of mass  $M_1$ , one spin-2 physical particle of mass  $M_2$ , one nonpropagating graviton and one scalar physical particle of mass  $m_0 = \sqrt{\frac{3b}{2}}$ ; the model is nontachyonic and tree nonunitary. The other models are tachyonic.

*Comment* : The topological term does not cure the nonunitarity of pure  $R + R_{\mu\nu}^2$  gravity.

- *Pure  $R + R^2$  Gravity* ( $\beta = 0, M^2 = \infty$ )

$$SP = \frac{T^{\mu\nu}T_{\mu\nu} - \frac{1}{2}T^2}{ak^2} + \frac{\frac{1}{2}T^2}{a(k^2 - \frac{\alpha\kappa^2}{2})}$$

$a = +1$  and  $\alpha > 0$ : One scalar physical particle of mass  $m_0 = \sqrt{\frac{\alpha\kappa^2}{2}}$  and one nonpropagating graviton; the model is nontachyonic and tree unitary.

$a = -1$  and  $\alpha > 0$ : One scalar ghost of mass  $m_0 = \sqrt{\frac{\alpha\kappa^2}{2}}$  and one nonpropagating graviton; the system is nontachyonic and tree nonunitary.

The remaining systems are tachyonic.

*Comment* :  $R + R^2$  gravity is tree unitary if and only if the sign of the Einstein's term is the conventional one.

- *Topological  $R + R^2$  Gravity* ( $\beta = 0$ )

$$SP = -\frac{T^{\mu\nu}T_{\mu\nu} - \frac{1}{2}T^2}{a(k^2 - a^2M^2)} + \frac{T^{\mu\nu}T_{\mu\nu} - \frac{1}{2}T^2}{ak^2} + \frac{\frac{1}{2}T^2}{a(k^2 - \frac{\alpha\kappa^2}{2})}$$

$a = -1$  and  $\alpha > 0$ : One spin-2 physical particle of mass  $M$ , one non-propagating graviton and a scalar ghost of mass  $m_0 = \sqrt{\frac{\alpha\kappa^2}{2}}$ ; the system is nontachyonic and nonunitary at the tree level.

$a = +1$  and  $\alpha > 0$ : One spin-2 ghost of mass  $M$  and a massive scalar physical particle of mass  $m_0 = \sqrt{\frac{\alpha\kappa^2}{2}}$ ; the model is nontachyonic and tree nonunitary. The other models are nonphysical due to the complex masses.

*Comment* : The topological term spoils the tree unitarity of the innocuous pure  $R + R^2$  gravity.

- *Higher-Derivative Gravity* ( $M^2 = \infty$ )

$$SP = -\frac{T^{\mu\nu}T_{\mu\nu} - \frac{1}{2}T^2}{a(k^2 + \frac{2a}{b})} + \frac{T^{\mu\nu}T_{\mu\nu} - \frac{1}{2}T^2}{ak^2} + \frac{\frac{1}{2}T^2}{a(k^2 - m^2)}$$

$a = +1$ ,  $b < 0$  and  $\frac{3}{2} + 4c < 0$ : One spin-2 ghost of mass  $m_2 = \sqrt{\frac{-2}{b}}$ , one nonpropagating graviton and one scalar physical particle of mass  $m$ ; the system is nontachyonic and nonunitary at the tree level.

$a = -1$ ,  $b > 0$  and  $\frac{3}{2} + 4c > 0$ : One spin-2 physical particle of mass  $m_2 = \sqrt{\frac{2}{b}}$  and one scalar ghost of mass  $m$ ; the model is nontachyonic and tree nonunitary.

The remaining models are tachyonic.

*Comment* : Higher-derivative gravity is nonunitary for both choices of the sign of the Einstein's term .

To conclude we remark that the enlargement of massive gravitational models via the topological term is a complete nonsense. On the one hand, it does not cure the nonunitarity of massive nonunitary systems (HDG,  $R + R_{\mu\nu}^2$  gravity). On the other hand, it spoils the unitarity of originally unitary masive models (FPG,  $R + R^2$  gravity).

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